INTRODUCTION TO QUANTUM MECHANICS, QUANTUM STATISTICS AND FIELD THEORY

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SUBJECT FUNDAMENTALS

The course aims to provide a presentation of main concepts of quantum mechanics and to discuss several applications of it, relevant for the future study of -- among others -- solid state, many-body theory and statistical mechanics.

EXPECTED LEARNING OUTCOMES

The students will develop a deep understanding of notions and concepts of non relativistic quantum mechanics and of a few special topics of an advanced nature:
the propagator and the formulation of quantum mechanics with the Feynman path integral; identical particles and second quantization; quantum circuits and Grover's algorithm.

PREREQUISITES/ASSUMED KNOWLEDGE

The course is self-contained - having followed a basic course of quantum mechanics is useful, but not strictly necessary.
At variance, the knowledge of linear algebra concepts, including the theory of vector spaces and the diagonalization of matrices, is required.

CONTENTS

1. Main concepts of quantum mechanics: the Stern-Gerlach interferometer.
2. Vector spaces of IBRA> and < KETI; linear, conjugate and Hermitian operators.
3. Eigenvectors and eigenvalues of operators; physical observables as Hermitian operators; discrete and continuous representations; the Dirac delta.
4. Eigenvectors and eigenvalues of the momentum, uncertainty principle.
5. Schroedinger equation, conserved quantities and stationary states.
6. Time evolution in quantum mechanics.
7. Reminder on one-dimensional problems: piecewise constant potentials, transmission-reflection coefficients, harmonic oscillator, delta potentials.
8. Eigenvalues and eigenvectors of angular momentum and kinetic energy.
9. Schroedinger equation in three dimensions; central potentials.
10. Problems treated with matrix techniques: harmonic oscillator, angular momentum, hydrogen atom.
11. Definition of the propagator and formulation of quantum mechanics using path integrals.
12. Variational techniques; application to time-dependent problems.
13. Time independent perturbation theory.
14. Time dependent perturbation theory, the transition probability per unit time, the Fermi golden rule.
15. Scattering theory; the scattering length.
16. Symmetries in quantum mechanics.
17. Combination of angular momenta in quantum mechanics.

18. Identical particles in quantum mechanics; fermions and bosons; construction of a wave function for a system of N identical particles; Slater determinant.
19. Two bosons in one-dimension interacting via a delta potential.
20. Elements of second quantization.
21. Mean-field equations for weakly interacting bosons; the non-linear Schroedinger equation.
22. Coherent states.
23. Basic notions on quantum circuits; Grover's algorithm.

DELIVERY MODES

Frontal lectures. Problem sessions on several applications of quantum mechanics, including the numerical solution of the Schroedinger equation via matrix techniques and pseudospectral methods.

TEXTS, READINGS, HANDOUTS AND OTHER LEARNING RESOURCES

J.J. Sakurai, Modern Quantum Mechanics (Addison Wesley) or Meccanica Quantistica Moderna (Zanichelli).

R. Shankar, Principles of Quantum Mechanics (Plenum Press).

M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press).

ASSESSMENT AND GRADING CRITERIA

The examination will be based on 2 midterm written tests and a final oral test.

To have access to the final oral test, a small numerical project has to be worked out, writing a code, and discussed. The final mark is provided by the average of the written (50%) and oral (50%) parts