INTRODUCTION TO QUANTUM MECHANICS, QUANTUM STATISTICS AND FIELD THEORY

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SUBJECT FUNDAMENTALS

The course aims to provide a presentation of the main concepts of quantum mechanics and to discuss several applications of it, relevant for the future study of – among others – solid state physics, many-body theory and statistical mechanics.

EXPECTED LEARNING OUTCOMES

The students will develop a deep understanding of notions and concepts of non-relativistic quantum mechanics and of a few special topics of an advanced nature: the propagator and the formulation of quantum mechanics with the Feynman path integral; identical particles; particles in periodic potentials and band structure in solids.

PREREQUISITES

The course is self-contained, having followed a basic course of quantum mechanics is useful, but not strictly necessary. At variance, the knowledge of linear algebra concepts, including the theory of vector spaces and the diagonalization of matrices, is required.

CONTENTS

1. The Rise of Quantum Mechanics (AR)

- The Black Body Problem (M. Planck, 1900- 1901)
- The Hypothesis of Photons and the Photoelectric Effect (A. Einstein, 1905)
- Optical Spectra of Atomic Elements (N. Bohr & A. Sommerfeld, 1913)
- The Quantization of Angular Momentum: the Stern-Gerlach Experiment (O. Stern & W. Gerlach, 1921-1922)

2. Mathematical Foundations: Vector Spaces (AR)

- Why Do We Need a New Formalism?
- ⟨bra\|'s and \|ket⟩'s: Definitions
- Operators in Quantum Mechanics

3. Hermitean Operators (AR)

- Definitions and Results
- Eigenvectors and Eigenvalues

4. Physical Observable (AR)

- Hermitean Operators and Physical Observables
- Functions of Observables
- Measuring Observables and Quantum averages: A physical Interpretation
- Commuting Observables

5. Representations in Quantum Mechanics (AR)

- Definitions
- The Dirac's δ-Function

- Representations of Quantum States: Observables with Discrete/Continuous Eigenspaces
- Representations of Operators
- Representations and Probabilities in QM
- Change of Representations
- Important Theorems on the Functions of Observables

6. Eigenvectors and Eigenvalues of the Momentum; Uncertainty Principle (AR)

7. Schrödinger Equation, Conserved Quantities and Stationary States (AR)

8. Time evolution in Quantum Mechanics (AR)

9. Reminder on one-dimensional problems (AR)

- Piecewise-Constant Potentials
- Transmission-Reflection Coefficients
- Harmonic Oscillator
- δ-potentials

10. Eigenvalues and eigenvectors of Angular Momentum and Kinetic Energy (AR)

11. Schrödinger Equation in three dimensions; Central Potentials (AR)

12. Problems treated with Matrix Technique (GG)

- Harmonic Oscillator
- Angular Momentum
- Hydrogen Atom

13. Definition of the Propagator and Formulation of Quantum Mechanics Using Path Integrals (GG)

14. Variational Techniques: Applications to time-dependent problems (AR)

15. Time-independent & Time-dependent perturbations theories (AR)

- The transition probability per unit time
- The Fermi’s golden rule

16. Scattering theory (AR)

17. Identical Particles in Quantum Mechanics (AR)

- Fermions & Bosons
- Construction of a Wave function for a system of N identical particles
- Slater determinant

18. Symmetries in Quantum Mechanics (GG)

19. Particles in periodic potentials: Bloch's Theorem & Band Structure (GG)

20. Particles in periodic potentials: Approximations & Examples (GG)

21. The density Operator (AR)

DELIVERY MODES

Frontal lectures, including problems sessions on several applications of quantum mechanics.

SUGGESTED REFERENCES

J.J. Sakurai, *Modern Quantum Mechanics* (Addison Wesley)

R. Shankar, *Principles of Quantum Mechanics* (Plenum Press).

P.A.M. Dirac, *The Principles of Quantum Mechanics* (Oxford University Press).

ASSESSMENT AND GRADING CRITERIA.

The examination will be based on 2 midterm written tests and a final oral test. To have access to the final oral test, a small numerical project has to be worked out, writing a code, and discussed. The final mark is provided by the average of the written (50%) and oral (50%) parts.